__Scientific report__

on the implementation of the project during January – December 2012

The team: project director, two senior researchers, two PhD students. Ana Agore defended her PhD thesis on October 1, 2012, with highest honors. Costel Bontea started his PhD studies on October 1, 2011, jointly at UB si Free Univ. Brussel. His training visits at the partner institution during 2012 were exclusively funded from the project funds.

Summary of the 2012 scientific report: 11 papers published/accepted, 10 of them in ISI ranked journals - the impact factor (IF) for 2011 is indicated. Another 6 papers are submitted for publication, all to ISI ranked. There were 5 presentations at international conferences and 2 workshop/colloquium presentations.

Published/accepted papers during 2012 (they all acknowledge the grant support):

[ACM1] A.L. Agore, S. Caenepeel, G. Militaru – The center of the category of bimodules and descent data for non-commutative rings, *J. Algebra Appl.* 11 (2012), 1-17. (IF 2011: 0.483)

[M2] G. Militaru – Representable functors for corings, *Comm. Algebra* 40 (2012), no.5, 1766-1796. (IF 2011: 0.347)

[AM3] A. L. Agore and G. Militaru – Schreier type theorems for bicrossed products, *Cent. Eur. J. Math.* 10 (2012), no. 2, 722-739. (IF 2011: 0.440)

[A6] A.L Agore – Coquasitriangular structures for extensions of Hopf algebras, *Glasgow Math. J.,* 55(2013) , 201-215, (IF 2011: 0.571)

[B15] S. Burciu, S. Natale - Fusion rules of equivariantizations of fusion categories, accepted in *Journal of Mathematical Physics.*- (IF 2011: 1.291)

[ABM4] A. L. Agore, C. G. Bontea, and G. Militaru – Classifying bicrossed products of Hopf algebras, accepted in *Algebr. Represent. Theory,* DOI: 10.1007/s10468-012-9396-5 (IF 2011: 0.595).

[ACM5] A.L. Agore, S. Caenepeel, G. Militaru – Braidings on the category of bimodules, Azumaya algebras and epimorphisms of rings, accepted in *Applied Cat. Structures* - DOI: 10.1007/s10485-012-9294-3, (IF 2011: 0.600)

[A7] A.L. Agore – Crossed product of Hopf algebras, accepted in *Comm. Algebra* (IF 2011: 0,347)

[ABM8] A. L. Agore, C. G. Bontea, and G. Militaru – Classifying coalgebra split extensions of Hopf algebras, accepted in *J. Algebra Appl.* - DOI: 10.1142/S0219498812502271, (IF 2011: 0.483)

[B9] S. Burciu – Subgroups of odd depth - a necessary condition, accepted in *Czechoslovak Math. J.* (IF 2011: 0.262)

[AM10] A. L. Agore, G. Militaru – Unified products and split extensions of Hopf algebras, accepted in *AMS Contemporary Math* – to appear in 2013.

Papers submitted for publication during 2012 (they all acknowledge grant support):

[ABM11] A. L. Agore, C. G. Bontea, and G. Militaru – The classification of all crossed products $H_4 \# k[C_n]$, submitted to *Internat. J. Algebra Comp.* (IF 2011: 0.453)

[AM12] A. L. Agore, G. Militaru – Bicrossed descent theory of exact factorizations and the number of types of groups of finite order, submitted to *J. Algebra* (IF 2011: 0.613)

[B13] S. Burciu – On coideal subalgebras of cocentral Kac algebras and a generalization of Wall's conjecture, submitted to *J. Algebra,* revision R1 was completed (IF 2011: 0.613)

[B14] S. Burciu – On the irreducible representations of generalized quantum doubles, submitted to *Adv. Math.* (IF 2011: 1.177)

[B16] C. G. Bontea – Classifying bicrossed products of two Sweedler's Hopf algebras, submitted to *Cent. Eur. J. Math.* (IF 2011: 0.440)

[CI17] V. Chari, B. Ion – BGG reciprocity for current algebras, submitted to *Compositio Math.* (IF 2011: 1.187)

Dissemination of results (talks given by project members):

[C1] G. Militaru - *Classifying bicrossed products of quantum groups,* Algebra Geometry Mathematical Physics, Brno, September 2012. Talk at international conference.
[C2] G. Militaru - *Extending structures: the level of groups,* Groups and their actions, Bedlewo, July 2012. Talk at international conference.
[C3] A.L. Agore - *Bicrossed descent theory of exact factorizations and the number of types of groups of finite order*, Groups and their actions, Bedlewo, July 2012. Talk at international conference.
[C4] C. Bontea - *Classifying crossed product of quantum groups,* Algebra Geometry Mathematical Physics (AGMP), Brno, September 2012. Talk at international conference.
[C5] A. L. Agore - *Deformations and descent type theory for Hopf algebras*, Algebra Geometry Mathematical Physics, Brno, September 2012. Talk at international conference.
[C6] G. Militaru - *Classifying bicrossed products. Deformations and descent type theory for quantum groups*, Hopf Algebra Workshop, Brussel VUB, 19 martie 2012.

[C7] B. Ion – *The Hopf-Poincare-Birkhoff-Witt theorem*, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA. Colloquium, February 2012.

Summary of scientific results:

The content of [ACM1] , [AM3], [ACM5] and [AM10] was detailed in the 2011 report as these papers were submitted to publication during 2011. The paper [M2], part of **objective II, problem 2b),** is studying four problems concerning representable functors. Let R and S be two rings, C a R-coring and ${}^{C}\mathcal{M}$ the category of left C-comodules. It is shown that the category of all representable functors ${}^{C}\mathcal{M} \rightarrow {}_{S}\mathcal{M}$ is the opossed category of ${}^{C}\mathcal{M}_{S}$. Furthermore, for a (S,R)-bimodule U there are given neccesary and sufficient conditions for the functor $U\otimes_R -: {}^{C}\mathcal{M} \rightarrow {}_{S}\mathcal{M}$ to be representable, equivalence of categories, separable, or Frobenius. The paper [ABM4] contributes to the **first objective** of the proposal and it approaches **problem 1b)**, more precisely, the classification up to isomorphism stabilizing A of the Hopf algebras the factorize through two given Hopf algebras A and H. Equivalently, there are classified up to a left A-linear Hopf algebra isomorphism all the bicrossed products between A and H. It is shown that these objects are classified by a cohomological object $\mathcal{H}^2$(A,H). In the construction of $\mathcal{H}^2$(A,H) the main role is played by certain elements of the product $\mathcal{CoZ}$(H,A) x $\mathcal{A}ut_{\rm CoAlg}$ (H), where $\mathcal{CoZ}$(H,A) is the set of unitary cocentral maps H → A and $\mathcal{A}ut_{\rm CoAlg}$ (H) is the set of unitary colagebra automorfisms H → H. Among several other applications and examples, there are described by generators and relations and classified all the bicrossed products between H_4 and $k[C_n]$. The number of isomorphism types of such quantum groups is determined by using Dirichlet's theorem. As a consequence, the automorphism group of these quantum groups is also determined. The paper [A6] contributes to the **first objective**, **problema 1a)**. The main theorem establishes a bijective correspondence between the set of all coquasitriangular structures on an arbitrary unified product of A and H and a data set (p, tau, u, v) that correspond to the components of the unified product. As an application necessary and sufficient conditions are obtained for the generalized quantum double introduced by Majid to be a coquasitriangular Hopf algebra. Also, explicit examples are presented. The paper [A7] is contributing to the **first objiective**, **problema 1a)**. Crossed products between Hopf algebras are studied. A Hopf algebra E is isomorphic to a crossed product beween two Hopf algebras A and H if and only if E factorizes through a normal Hopf subalgebra A and a subcoalgebra H. The main properties of crossed products are studied, such as: the universality of the construction, the existence of integrals, the commutativity, the involutivity. From a categorical point of view, there are described all the braided structures on the category of comodules over a crossed product, in terms of the components of the crossed product. The paper [ABM8] contributes to the **first objective** of the proposal and it approaches **problem 1a)**, more precisely, the classification of all the crossed products between two given Hopf algebras A and H. It is shown that any extension of A through H that splits in the category of coalgebras is equivalent to a crossed product of A and H associated to a crossed system of Hopf algebras. The problem of classifying the extensions of A through H that split in the category of Hopf algebras is thus reduced to a purely computational problem. It is shown that the set of crossed systems associated to the pair (A, H_4), where H_4 is Sweedler's 4-dimensional Hopf algebra, is in bijection with the set $\mathcal{ZP}(A)$ of all central primitive elements of A (the result holds for any Hopf algebra A). Furhtermore, the set of equivalence classes of crossed products of A and H_4 is in bijection with a quotient set of $\mathcal{ZP}(A)$. To illustrate, several explicit examples for A are considered, among them the polynomial Hopf algebra and the p-dimensional semisimple Hopf algebra. The paper [B9], part of the **first objective, problem** 2d), contains a combinatorial condition for a subgroup with trivial core to be an odd depth subgroup. The condition is presented as a the rank maximality of a certain 0-1 matrix. The paper [ABM11] is a continuation of [ABM8] and is contributing to the **first objective**, **problema 1a)**. Using the method developed in [ABM8] a classification of all the crossed products of H_4 and $k[C_n]$ is obtained, where the latter Hopf algebra is the group Hopf algebra associated to the cyclic group with n elements, C_n. The conclusion is that the set of crossed systems associated to the pair (H_4, $k[C_n]$) is in bijection to the set $\mathcal{CS}(n, k)$ of all the pairs (t, λ), where $t : C_n \rightarrow C_2$ is a unitary function and λ is a root of ±1 (the sign being determined by the signature of t), and the set of quivalence classes of crossed products of H_4 and $k[C_n]$ is in bijective correspondence with a certain quotient set of $\mathcal{CS}(n, k)$. The paper [AM12] is part of the **first objective**, **problem 1b)**. If A is a subgroup of G and H is a factorization A-form of G then it is shown that aany other factorization A-form H of G is isomorphic to the deformation H_r, where r : H → A is a descent map. Similarly as in the case of Hopf algebras, it is shown that there exists an isomorphism between the set of isomorphism types of factorization A-forms of G and the cohomological object $\mathcal{HA}$(A,G | ►, ◄). The main application presented is the fact that any group with n elements is isomorphic to a r-deformation of the cyclic group C_n and an explicit formula for the group structure is obtained. The paper [B13] is part of the **first objective, problema 1c)**. It is shown that any coideal subalgebra of a finite dimensional Hopf algebra is a cyclic module over the dual Hopf algebra. Using this description, the coideal subalgebras of a Kac type semisimple Hopf algebra were enumerated, thus contributing to the theory of semisimple Hopf algebras. This enumeration of the coideal subalgebras allows the proof of a conjecture formulated by R. Guralnick si F. Xu. In the paper [B14], part of the **second objective, problem 2e)**, a description of the irreducible representations of the quantum doubles associated to a skew pairing of Hopf algebras is obtained. In particular, a description of the irreducible representations of the semisimple Drinfeld quantum doubles is obtained. This description is obtained by making use of the Clifford theory for semisimple Hopf algebras which was developed in a previous paper. It is also shown that the Grothendieck ring of these generalized quantum doubles has a structure that is similat to that of the rings that are associated to the Green functors. In the paper [B15], part of the **second objective, problem 2d),** the fusion rules of a equivariantization of a fusion category C under the action of a finite group G are determined. These fusion rules are described in terms of the fusion rules of C and by a certain group theoretical part of the action of G on C. As application a formula is obtained for the fusion rules for the equivariantization of pointed fusion category. In turn, this determines the fusion rules of any braided group theoretical fusion category. The fusion rules for the category of finite dimensional representations of a twisted Drinfeld double associated to a finite group are completely determined by using the above description. The paper [B16] part of the **first objective**, **problem 1b)**, represents an application of the methods developed in [ABM10] to a concrete case, more precisely to the classification of crossed products between two Sweedler Hopf algebras. It is shown that, except for the trivial pair, the matched pairs associated to (H_4, H_4) are parametrized by the elemets of the base field *k*. To each element λ in *k* it corresponds a bicrossed product, $\mathcal{H}_{4,\lambda}$, which is a 16-dimensional, pointed, unimodular, non-semisimple Hopf algebra. The isomorphism classes of such Hopf algebras are represented by the tensor product between H_4 and H_4, $\mathcal{H}_{4,0}$ and $\mathcal{H}_{4,1}$, the latter being isomorphic, it turns out, to the Drinfeld double of H_4. The paper [CI17], contributes to the **second objective, problem 2b)**, and is concerned with the existence of a duality in a category of representations of current Lie algebras, duality that is akin to the one discovered by Bernstein-Gelfand-Gelfand for the category *O* associated to a semisimple Lie algebra. The main result, (conjectured by Chari et al. in 2011) establishes the equality between the multiplicity of a simple object in a local Weyl module and the multiplicity of a global Weyl module in the projective cover of a simple object. In principle, the target category is the category of finite dimensional representations for the quantized enveloping algebra of an affine Lie. The study of this category was reduced to the study of the category of graded weight modules with finite dimensional homogeneous components for the so-called current Lie algebras. In [CI17] we identify this category with the category of weight modules for the special maximal parabolic subalgebra of an affine Lie algebra. The characters of the Weyl modules are identified with certain limits of Macdonald polynomials.

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Project director,

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