Raport stiintific sintetic

privind implementarea proiectului in etapa 2014

Abstract: During the 2014 stage, 8 papers were published/accepted for publication by ISI ranked journals. Another 5 papers were submitted for publication by the members of the research team. There were 3 presentations at international conferences.

Summary of the scientific results obtained:

[1] V. Chari, B. Ion – BGG reciprocity for current algebras, accepted for publication in *Compositio Math.* (AIS=2.33, IF=1.18)

This paper contributes to the second objective, problem 2b), and is concerned with the existence of a duality in a category of representations of current Lie algebras, duality that is akin to the one discovered by Bernstein-Gelfand-Gelfand for the category *O* associated to a semisimple Lie algebra. The main result, (conjectured by Chari et al. in 2011) establishes the equality between the multiplicity of a simple object in a local Weyl module and the multiplicity of a global Weyl module in the projective cover of a simple object. In principle, the target category is the category of finite dimensional representations for the quantized enveloping algebra of an affine Lie. The study of this category was reduced to the study of the category of graded weight modules with finite dimensional homogeneous components for the so-called current Lie algebras. We identify this category with the category of weight modules for the special maximal parabolic subalgebra of an affine Lie algebra. The characters of the Weyl modules are identified with certain limits of Macdonald polynomials.

[2] V. Chari, B. Ion, D. Kus – Weyl modules for the hyperspecial current algebras, accepted for publication in *Int. Math. Res. Notices* (AIS=1.55, IF=1.01)

This paper contains most of the technical results needed in [1] and contributes to the second objective, Problem 2b), of our grant proposal. In particular in [2], the theory of local and global Weyl modules is constructed for twisted current algebras. There have been many attempts in the literature to extend the Weyl module theory to this context but all the results obtained so far fail in constructing such a theory for the case of the twisted algebras of type $A^{(2)}_{2n}$, which is the most complex example of such algebras. The difficulty in this case consists in the new phenomena which appears due to the fact that this algebra contains subalgebras of rank 1 of a new type of isomorphism. In [2] we construct a theory which works for all cases. As in [1], the main conceptual advantage is the realization of twisted current algebras as parabolic algebras in twisted affine Lie algebras. The main technical contribution is a Garland type lemma for Lie algebras of type $A^{(2)}_{2}$. For obtaining this result we use the structure of the PBW basis for algebras of this type. From this point of

view, [2] contributes also to the second objective, Problem 2c). Another important result from [2] is that the global Weyl modules are free over a certain polynomial algebra.

Moreover, the local Weyl modules are characterized as special Demazure modules. It is also worth pointing out that this result is true for twisted current algebras but not in the case of classical current algebras.

[3] A.L. Agore, Free Poisson Hopf algebras generated by coalgebras, *J. Math. Phys.* 10 (2014), 083502. (IF = 1.291)

This paper which contributes to the second objective, Problem 2b), of our grant proposal studies the category of Poisson Hopf algebras. A Poisson Hopf algebra is both a Poisson algebra and a Hopf algebra such that the comultiplication and the counit are morphisms of Poisson algebras. Such objects are situated at the border between Poisson geometry and quantum groups. In this paper we study some categorical properties of Poisson Hopf algebras and Poisson bialgebras. The analogue of Takeuchi's free Hopf algebra in the setting of Poisson Hopf algebras is constructed. More precisely, we prove that there exists a free Poisson Hopf algebra on any coalgebra or, equivalently that the forgetful functor from the category of Poisson Hopf algebras to the category of coalgebras has a left adjoint. In particular, we also prove that the category of Poisson Hopf algebras is a reflective subcategory of the category of Poisson bialgebras. Along the way we describe coproducts and coequalizers in the category of Poisson Hopf algebras, therefore showing that the latter category is cocomplete.

[4] A.L. Agore, Classifying complements for associative algebras, *Linear Algebra Appl.* 446 (2014), 345-355. (IF=0.974)

This paper is part of the first objective, Problems 1a) and 1b), of our grant proposal and treats the classifying complements problem (CCP) in the context of associative algebras. Let $A \subset E$ be a given extension of algebras. If X is a given A complement of E then we can all complements of A in E as follows: any A-complement of E is isomorphic to an r-deformation of X. In other words, exactly as in the case of Hopf algebras, Lie algebras or Leibniz algebras, given X an A-complement of E all the other A-complements of E are deformations of the algebra X by certain maps $r : X \to A$ associated with the canonical matched pair which arises from the factorization E = A + X. The theoretical answer to the (CCP) is by explicitly constructing a cohomological type object $HA^2(X,A|(\triangleright, \triangleleft, \leftharpoonup, \rightharpoonup))$ which parameterizes all A complements of E. We introduce the factorization index $[E : A]^f$ of a given extension $A \subset E$ as the cardinal of the (possibly empty) isomorphism classes of all A-complements. Moreover, we prove that the factorization index is computed by the formula: $[E : A]^f = |HA^2(X,A|(\triangleright, \triangleleft, \leftharpoonup, \rightharpoonup))|$.Several explicit examples are provided. More precisely, we

indicate associative algebra extensions whose factorization index is 1, 2 or 3. The paper ends with an extension of index at least 4.

[5] A.L. Agore, G. Bontea, G. Militaru, The classification of all crossed products H_{4} # k[C_{n}], *Symmetry Integr. Geom.* 10 (2014), 049, 12 pages. (IF=1.071)

This paper is part of the first objective, Problem 1a), of our grant proposal and is a continuation of our previous paper A.L. Agore, G. Bontea, G. Militaru, *Classifying coalgebra split extensions of Hopf algebras*, J. Algebra App. 12 (2013), 1 – 24. More precisely, using the computational approach introduced in the aforementioned paper, we classify all coalgebra split extensions of the Sweedler's Hopf algebra H_4 by the group Hopf algebra of the cyclic group of order n, $k[C_n]$, i.e. all Hopf algebras E that fit into a sequence $H_4 \mapsto E \xrightarrow{\pi} k[C_n]$ such that $\pi : E \to k[C_n]$ splits as a coalgebra map and $H_4 \cong E^{co(k[C_n])}$. Equivalently, we classify all crossed products of Hopf algebras $H_4 \# k[C_n]$ by explicitly computing two classifying objects: the cohomological group $H^2(k[C_n], H_4)$ and $Crp(k[C_n], H_4)$:= the set of types of isomorphisms of all crossed products $H_4 \# k[C_n]$. The first step taken is to compute the set of all crossed systems of Hopf algebras $(H_4, k[C_n], \triangleright, f)$ and to describe the corresponding crossed products. This first computational part of our strategy is quite laborious considering the large number of axioms that need to be fulfilled by the pairs $(\triangleright, f)$ in order to provide a crossed system of Hopf algebras $(H_4, k[C_n], \triangleright, f)$. Explicit classification results are proven. More precisely, all crossed products $H_4 \# k[C_n]$ are described by generators and relations and classified: they are 4n -dimensional quantum groups $H_{4n,\lambda,t}$, parameterized by the set of all pairs (λ, t) consisting of an arbitrary unitary map $\lambda : C_n \to C_2$ and an n-th root λ of ± 1. As an application, the group of Hopf algebra automorphisms of $H_{4n,\lambda,t}$ is explicitly described.

[6] G. Militaru – The global extension problem, co-flag and metabelian Leibniz algebras, Linear Multilinear Algebra 63(3) (2015), 601-621. (IF=0.727)

This paper is part of the first objective, Problem 1a), as well as of the second objective, Problem 2b), of our grant proposal. Let G be a Leibniz algebra, E a vector space and p: E → G an epimorphism of vector spaces. The main result of the paper describes and classifies all Leibniz algebra structures that can be defined on E such that p becomes a Leibniz algebra map. In this way we obtain a theoretical answer to what we have called 'the global extension problem' as a dual to ' the extending structures problem' from the first objective of the project. All these structures are classified by a global coomological object which we construct. Moreover we prove that this coomological object is a coproduct of all local cohomological
Objects in which the representations of the Leibniz algebra play a key role. In particular, the classical second cohomology group of Loday and Pirashvili appears as the most elementary

piece among all components of this global classification object. Several applications are obtained for co-flag and metabelian Leibniz algebras.

[7] A.L. Agore, G. Militaru, Bicrossed products, matched pair deformations and the factorization index for Lie algebras*, Symmetry Integr. Geom.* 10 (2014), 065, 16 pages (IF=1.071)

This paper continues the investigation started in our previous papers A.L. Agore, G. Militaru, ''Classifying complements for Hopf algebras and Lie algebras'', J. Algebra 391 (2013) si respectiv A.L. Agore, G. Militaru, ''Extending structures for Lie algebras'', Monatshefte fur Mathematik 174 (2014) related to the factorization problem for Lie algebras and its converse, the classifying complements problem. Our starting point is a result from [10] which describes all Lie algebras L that contain a given Lie algebra h as a subalgebra of codimension 1 over an arbitrary field k: the set of all such Lie algebras L is parameterized by the space TwDer(h) of twisted derivations of h. For a perfect Lie algebra h we classify all Lie algebras containing h as a subalgebra of codimension 1. The automorphism groups of such Lie algebras are fully determined as subgroups of the semidirect product between h and (k^* x Aut_{Lie} (h)). For the special case of sympathetic Lie algebras h, we prove that, up to an isomorphism, there exists only one Lie algebra that contains h as a Lie subalgebra of codimension one, namely the direct product of h with the abelian Lie algebra and moreover Aut_{Lie} (k_0 x h) = k^* x Aut_{Lie} (h). In the non-perfect case the classification of these Lie algebras is a difficult task. Let l(2n + 1, k) be the Lie algebra with the bracket given by $[E_i, G] = E_i$, $[G, F_i] = F_i$, for all i = 1,…, n. We explicitly describe all Lie algebras containing l(2n + 1, k) as a subalgebra of codimension 1 by computing all possible bicrossed products k $\triangleright\!\!\triangleleft$ l(2n + 1, k). They are parameterized by a set of matrices M_n (k) ^{4} x k^{2n+2} which are explicitly determined. Several matched pair deformations of l(2n + 1, k) are described in order to compute the factorization index of some extensions of the type k $\subset$ k $\triangleright\!\!\triangleleft$ l(2n + 1, k). We provide an example of such extension having an infinite factorization index. We also consider the problem of establishing which properties of Lie algebras are preserves by the matched pair deformation. Several counterexamples are provided. The paper ends with an open question having as source of inspiration a similar result previously proved in the context of groups.

[8] S. Burciu – On coideal subalgebras of abelian cocentral extensions and a generalization of Wall's conjecture, J. Algebra Appl. 14 (2) 2015, 1550021 (IF=0.483).

This paper describes the left and right coideal subalgebras of a cocentral, abelian extension of Hopf algebras over an algebraically closed field of characteristic zero (the so-called ''Kac algebra''). The results of this paper are part of the first objective, Problem 1c) as well as of the second objective, Problem 2d), of our grant proposal. It is well-known by a result of S. Natale that cocentral extensions give rise to equivariantized fusion categories. Also, it is known that fusion subcategories of the category of representations of a semisimple Hopf algebra are described by left (or right) coideal subalgebras closed to the adjoint left or right action. The above description proves that any fusion subcategory of an equivariantization of pointed categories is also an equivariantization of pointed categories. Moreover, this allows us to formulate a Wall type conjecture (see R. Guralnick and F. Xu "On a subfactor generalization of Wall's conjecture" Journal of Algebra, 332, (1), 2011, 457-468.) for semisimple Hopf algebras. Finally, we give an affirmative answer to this conjecture in the setting of abelian cocentral extensions over the field of complex numbers.

Papers submitted for publication:

G. Militaru – *Metabelian associative algebras and bilinear forms*

A.L. Agore, G. Militaru - *Ito's theorem and metabelian Leibniz algebras*

A.L. Agore, G. Militaru - *Jacobi and Poisson algebras*

A.L. Agore, G. Militaru – *The extending structures problem for algebras*

A.L. Agore, G. Militaru – *The global extension problem, crossed products and co-flag non-commutative Poisson algebras*

11.11.2014 Project director,

Prof. dr. Gigel Militaru